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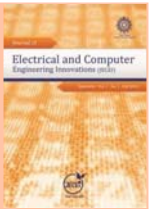

A Comparison of Different Control Design Methods for the Linearized CSTR Temperature Model

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ABSTRACT

Continuous Stirred Tank Reactor (CSTR) has particular importance in chemical industry. CSTR has usually a nonlinear behavior which makes it difficult to control. The reactor has two parameters: the concentration and temperature of mixture both of which are uncertain. This case of CSTR has large disturbance in domain. In order for disturbance rejection, a controller has to be designed. In this paper, for modeling the CSTR system, first, the *PI* and *PID* controllers are designed by two methods, the automatic with Matlab Simulink and Ziegler-Nichols *(Z-N)* method. Then, reset control is replaced and tuned by their parameters. The main aim of this work is to compare the output responses (temperatures) of controllers with each other. In this work a reset controller is proposed for the thermal reactor model. Due to complexity of control of this plant, different design methods should be evaluated for disturbance rejection and input tracking. The results show that the reset controller is better than the *PI* controller in disturbance elimination. Finally, controller's output response is investigated for improvement in disturbance rejection and change in the set-point.

1. INTRODUCTION

A CSTR refers to a continuous stirred tank reactor which its significant usage is in chemical industry. The type of nonlinearity in system and unpredictable behavior of system parameter makes it difficult to control. Within the efforts to design a proper controller for CSTR system, some methods are developed by electrical engineer, such as using fuzzy logic system [1], sliding mode control, adaptive state feedback, neural network and robust design methodology to control the CSTR system which has been proposed earlier by Chen and Dai [2, 3]. In this paper, different controllers have been designed for the linearized reactor model by using the $Z-N$ method, reset control and automatic tuning methods. Then their output responses have been compared in terms of disturbance rejection and set-point tracking.

In chemical processes, the liquid has to be heated and reached to the desired temperature in order that the chemical reaction takes place. The heating should not be done directly because it may bring temperature too high to the point which the liquid become flammable. Firstly, a liquid such as water has been headed to steam then passed through the radiator pipes and liquid becomes hot in the tank [4]; See Fig. 1.

A large amount of steam in tube causes the original liquid become warmer. The goal is temperature control of the output liquid from the tank. This type of process control is known as error-based control because the actuating signal is determined from the error between the actual and desired setting.

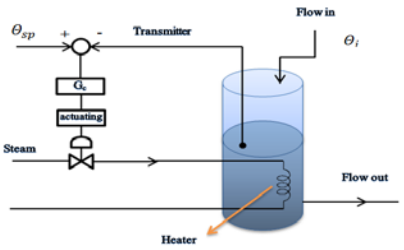

Figure 1: Schematic scheme of the temperature controller in a CSTR

The tank has three inputs: Reactant's flow rate F, input liquid temperature Θ_i and input steam w, when F, Θ_i are disturbance and uncontrollable, respectively.

The CSTR is an important process system in chemical industries. Due to the complex dynamic behavior, the control of that is so difficult. Therefore, finding a way to curb the nonlinear behavior is so precious.

In this work, firstly the process model is acquired and then linearized around the operating point. In the first step, the PI controller is designed and then the reset controller is replaced and tuned. We design a PI controller and then a reset controller for the CSTR temperature model in order to obtain limitations and advantages of the reset controller. In the next step, we design a PID controller and compare its output response with the other controllers.

The aim of this paper is indeed to present a comparison between the performance of reset and the other controllers. In this way, we describe by some proof how we can achieve to a better performance in the terms of set point tracking and disturbance rejection with a simple and low cost reset action than a PI and in some cases PID controllers for complex system.

Reset control is a Clegg integrator CI which was introduced in the first time by Clegg [5]. The CI consists of a linear integrator LI and a reset mechanism which reset the state of linear integrator to zero when its input disappears (Clegg 1958). It has the ability to overcome the limitations of the linear and linear time-invariant (LTI) control systems. In addition, a Clegg integrator has a similar magnitude-frequency response as a pure integrator, but with 51.9° less phase lag. It has a describing function given by $\frac{1.62}{j\omega}e^{j52^\circ}$. This favorable property helps to increase the phase margin of a system [6]. The advantage and disadvantage of a reset control have been discussed in many papers; see [4-9]. Note that the reset control doesn't lead to stabilization, but in fact it may destabilize a (LTI) feedback system. Thus it has to be used with care. In recent years, reset control systems are being used in a wide range of application, e.g., about closed loop stability [10] and stability with delay in reset systems [11, 12] which give suitable guidelines for designing the reset control solar collector field [13], temperature control in heat exchangers [14] and reset control of an industrial in-line pH process [15]. The $PI+CI$ consists of a PI compensator and CI including k_{reset}, τ_{reset}, ρ_{reset}, which is obtained according to equation (10).

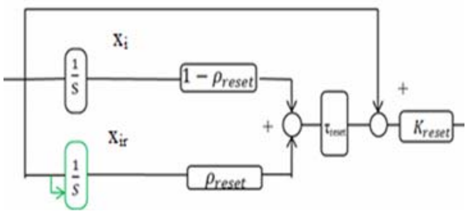

Figure 2: Block diagram of PI+CI

$k_p = k_{reset}$ is the proportional gain, e and v are error and control signal, respectively, τ_i is the integral time constant, x_i is the I-term state and x_{ir} is the CI-term state. The parameter $\rho_{reset} \in [0,1]$ is the reset ratio which calculates the CI to I term ratio. When $\rho_{reset} = 0$, the $PI+CI$ compensator is converted to a PI controller [5]. The $PI+CI$ basically consists of adding a Clegg integrator, CI, to a Proportional-integral, PI controller, with the goal of improving the closed loop response by using the nonlinear characteristic of this element. It turns out that by resetting a percentage of the integral term of a PI controller, a significant improvement can be obtained by considerably reducing overshoot percentage and settling time [16].

This paper is organized as follows: in section 1, CSTR and reset control are described. In next section, dynamic modeling of CSTR is introduced. Moreover Simulink PI, $(PI+CI)$ controllers are shown; in section 4, simulation results of the controllers are compared. Then comparison result with the PID controller from disturbance rejection and input tracking is presented. Section 6 is devoted to the investigation of error integrator creator in CSTR model and efficient of reset ratio on the error integral criterion. Finally, conclusions are drawn in section 7.

2. DYNAMIC MODELING OF CSTR

Energy balance equation is given by [4]:

$$V\rho C_p \frac{d\Theta}{dt} = F(t)\rho C_p \Theta_i(t) + UA(\Theta_s(t) - \Theta(t)) - F(t)\rho C_p \Theta(t) \qquad (1)$$

The heat transfer coefficient is $U = 1/R$, where R is the heater thermal resistance.

$$V\rho C_p \frac{d(\overline{\Theta}+\theta)}{dt} = (F + f(t))\rho C_p(\overline{\Theta}_i + \theta_i(t)) + UA(\overline{\Theta}_i + \theta_s(t) - \overline{\Theta} - \theta(t)) - (\overline{F} + f(t))\rho C_p(\overline{\Theta} + \theta(t)) \quad (2)$$

where $\overline{\Theta}_i$, $\overline{\Theta}_s$, $\overline{\Theta}$ and $\overline{F}$ are the operating points. The system has been linearized around the operating point.

The output to input transfer function and the disturbance after some computation are obtained as:

$$\theta(s) = \frac{k_f}{\tau s+1} f(s) + \frac{k_i}{\tau s+1}\theta_i(s) + \frac{k_s}{\tau s+1}\theta_s(t) \quad (3)$$

where the parameters k_f, k_i, k_s, τ are determined by:

$$\begin{aligned} k_f &= \frac{\rho C_p(\overline{\Theta}_i - \overline{\Theta})}{UA + \overline{F}\rho C_p} \\ k_i &= \frac{\overline{F}\rho C_p}{UA + \overline{F}\rho C_p} \\ k_s &= \frac{UA}{UA + \overline{F}\rho C_p} \\ \tau &= \frac{V\rho C_p}{UA + \overline{F}\rho C_p} \end{aligned} \quad (4)$$

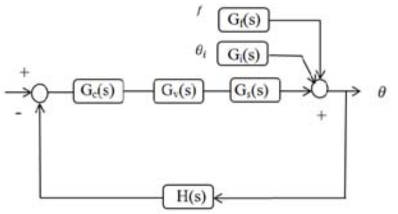

Figure 3: Block diagram of CSTR model

Dynamic sensor $H(s)$ is estimated as the following first-order system

$$\begin{aligned} H(s) &= \frac{k_t}{\tau_t s + 1} \\ G_f(s) &= \frac{k_f(\tau_s s+1)}{(\tau s+1)(\tau_s s+1) - k_s} \\ G_i(s) &= \frac{k_i(\tau_s s+1)}{(\tau s+1)(\tau_s s+1) - k_s} \\ G_s(s) &= \frac{k_w k_s}{(\tau s+1)(\tau_s s+1) - k_s}, G_v(s) = \frac{k_v}{\tau_v s+1} \end{aligned} \quad (5)$$

where $G_v(s)$ shows the valve and actuating dynamics. The CSTR of parameters are obtained according to Table (11) and equations (5).

$$\begin{aligned} &k_f = 2386^\circ Cs/m^3,\ k_i = 0.619^\circ C/^\circ C \\ &k_s = 0.381^\circ C/^\circ C,\ k_w = 142.8^\circ C/(kg/s) \\ &k_v = 0.024 kg/s.\%,\ \tau_s = 32s, \tau = 309.3s \\ &\overline{\Theta}_s = 108.8^\circ C \end{aligned} \quad (6)$$

3. SYNTHESIS OF THE CONTROLLERS

A. Simulation with a PI controller

A proportional-integral (PI) controller has two controller modes, combination of the P and I controller. When the integration constant is fixed, increasing the proportional constant will increase the control activity (aggressiveness) and correspondingly, decreasing the integration constant will increase the control activity and response rate but may lead to oscillations in the response.

The PI controller is given by:

$$PI(s) = k_p(1 + \frac{1}{\tau_i s}) \quad (7)$$

B. Simulation with a PI+CI controller

The $PI + CI$ is a PI compensator that consists of a PI compensator and a reset section (CI) with a new additional parameter ρ_{reset} .The $PI + CI$ can overcome PI compensator basic limitations which is one of its advantages in comparison with a PI compensator. Although the PID Compensator may have better performance, but it's more favorable to use the $PI + CI$ controller instead because D term will raise the cost of feedback.

The $PI + CI$ compensator has a transfer function including a proportional gain k_p, and an integral time constant τ_i.

$$(PI + CI)(j\omega) = k_p\left(\frac{j(\omega\tau_i + \frac{4\rho_{reset}}{\pi}) + 1}{j\omega\tau_i}\right) \quad (8)$$

A realization of the PI compensator in the state space is given by:

$$PI \begin{cases} \dot{x}(t) = e(t) \\ v(t) = \dfrac{k_p}{\tau_i} x(t) + ke(t) \end{cases} \quad (9)$$

The $PI + CI$ compensator has two terms; Clegg integrator (CI) in parallel with a PI controller. The CI term can improve the transient response and reduce the output response overshoot [5].

The structure of the $PI + CI$ is shown in Fig. 2. Tuned parameters of $PI + CI$ are obtained as:

$$\begin{aligned} &k_{reset} = 1.75k_p, \tau_{reset} = 1.25\tau_i, \rho_{reset} = 0.3\ Auto-Tun \\ &k_{reset} = 0.33k_p, \tau_{reset} = 0.125\tau_i, \rho_{reset} = 0.4\ Z-N \end{aligned} \quad (10)$$

4. SIMULATION RESULTS

In the systems under study, the disturbance f and output liquid temperature Θ_i are assumed as constants. Two methods are applied to design the controllers: 1. $Z-N$ method, 2. Auto tuning by Matlab Simulink.

The $PI+CI$ $Z-N$ method is a developed method of tuning a PID controller which is performed by setting the parameters of PI. Tuned parameters of PI (k_p,τ_i) and PID (k_p,τ_i,τ_d) controllers of CSTR model are shown according to Table (1).

TABLE 1
PI, PID CONTROLLER PARAMETERS WITH Z-N

Parameter	k_p	τ_i	τ_d
PI	4.75	227	-
PID	6.33	136	34

In second method, the controller gains are tuned using Matlab by launching the $(PI-PID)$ tuner; then the software automatically computes the controller parameters of CSTR model according to Table (2)

TABLE 2
PI, PID CONTROLLER PARAMETERS WITH AUTO TUNED METHOD

Parameter	k_p	τ_i	τ_d
PI	0.69	345	-
PID	0.78	41.71	61

Remarks:

The following acquired results show the controllers performance for the nonlinear model of CSTR:

A. Automatic tuned method: As shown in Fig. 4, the reset control can decrease the disturbance and eliminate it faster than the PI controller. The rise time has been decreased from 373 to 360 seconds, the settling time has been decreased from 1550 to 1100 seconds, the overshoot is also lower; see table (3). Therefore the reset controller response is better than the PI controller response.

B. $Z-N$ Process: although disturbance domain increases in the reset controller, but it does react faster. In this case, the reset compensator response is relatively better than the PI controller; see Fig. 5.

As shown in Fig. 5, the PI controller tuning with $Z-N$ method has more oscillations while the reset controller has reached steady state in a shorter time.

Fig. 6 and Fig. 7, show the control efforts, in this case the response is faster without undershoot but the in $Z-N$ method the PI controller is oscillatory and the reset control signal reaches to the steady state value very fast without oscillation.

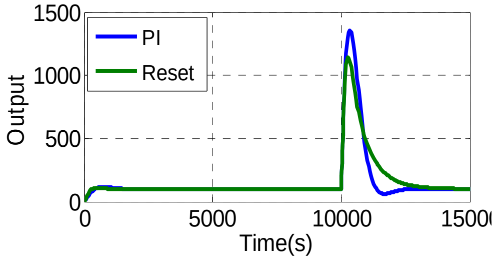

Figure 4: Step output response with tuned parameters

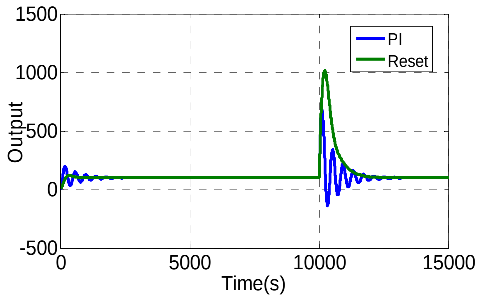

Figure 5: Step output response with Z-N method

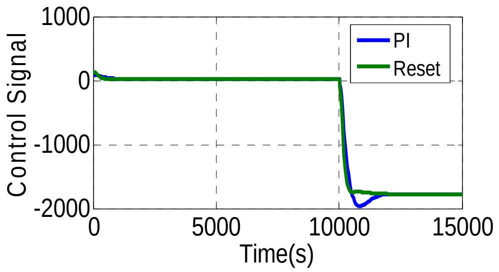

Figure 6: control action response (u) with tuned parameters

TABLE 3
RESPONSE DATA OF FIGURES (4) AND (5)

Data	PI $(Z-N)$	PI $(Auto-Tun)$	$PI-CI$
T_r (sec)	73.6	373	360
T_s (sec)	2250	1550	1100
Overshoot	76.1%	10%	4.5%

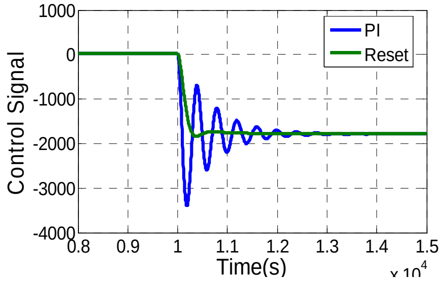

Figure 7: Control action response (u) with Z-N method

5. COMPARISON WITH THE PID CONTROLLER:

The *PID* controller includes proportional, integration and derivation terms which are defined by:

$$PID = k_p(1 + \frac{1}{\tau_i} + \tau_d) \tag{11}$$

In this section, *PID* controller is designed based on automatic tuning and $Z-N$. The parameters in tables (1) and (2) are applied to the *PID* controller. As shown in Fig. 10 and Fig. 11, the automatic tuning output response has more overshoot and lower rate than the reset and *PI* controllers. In the $Z-N$ method, the *PID* controller has lower overshoot and faster response. Simulation results and comparison between the controllers show that in the $Z-N$ process, the *PID* has better performance and the reset controller is also better than the *PI* controller in disturbance rejection.

A. Disturbance Rejection

Different controllers have been designed for the linearized reactor model using the $Z-N$ method, reset control and automatic tuning methods. Then their output responses have been compared in terms of disturbance rejection.

Thus, the reset controller is a suitable replacement for most cases which can overcome the *PI* controller fundamental limitations. The response data are summarized in table (4),

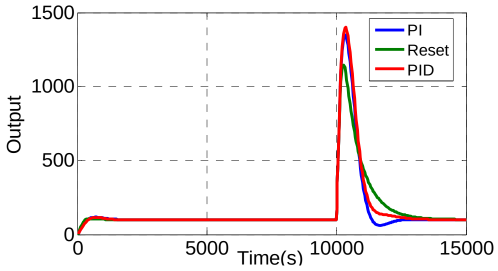

Figure 8: Output response with tuned parameters

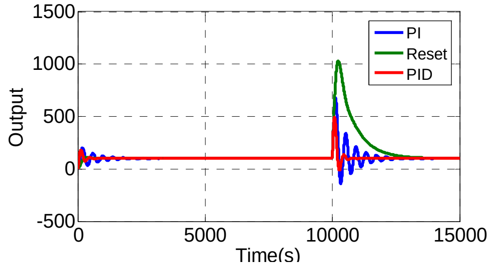

Figure 9: Output response with Z-N method

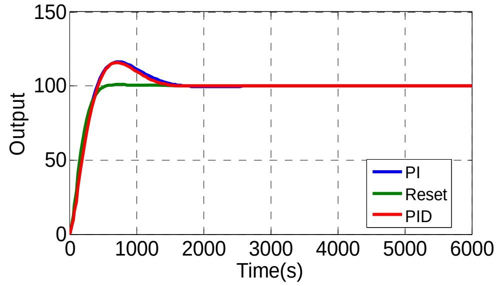

Figure 10: Output response with tuned parameters for change of setpoint

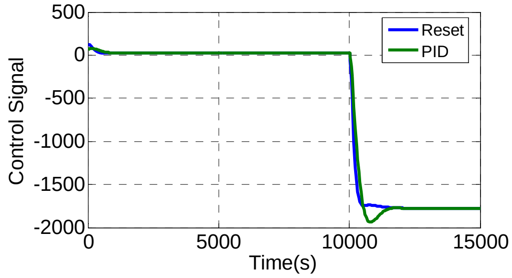

Figure 11: control action response (u) with tuning method

Table 4
Response Data of Figures (8) and (9)

Data	*PID* $(Z-N)$	*PID* $(Auto-Tun)$
T_r (sec)	57.4	384
T_s (sec)	597	1330
Overshoot	55.3%	8.5%

As shown in Fig. 11, the control action response (u) with tuning method has lower undershoot in the reset control mode.

B. Set-point tracking

In this part, a step of magnitude 100 is applied as the reference input. As shown in Fig. 12 and Fig. 13 in the automatic tuning method, the reset output response has less overshoot and rise time in comparison with PI and PID controllers as well as it has attained the steady state value in a shorter time. The control signal has lower settling time in spite of its higher initial value.

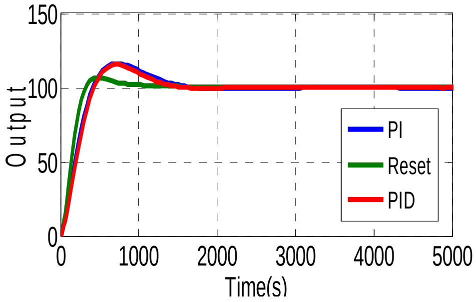

Figure 12: Output response with tuned parameters for change of setpoint

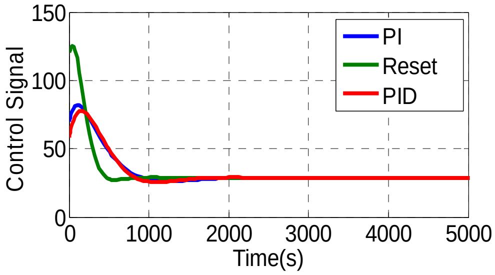

Figure 13: control action response (u) with Auto-tuned method for setpoint tracking

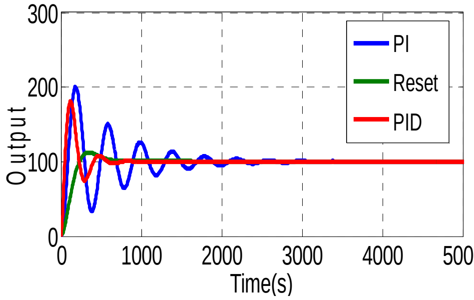

Figure 14: Output response with Z-N parameters for setpoint tracking

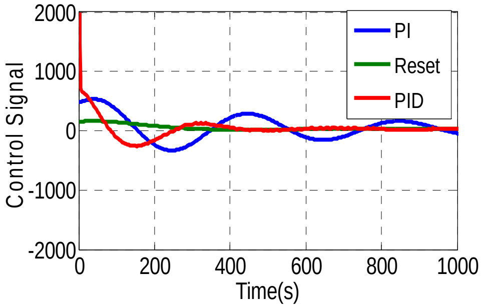

Figure 15: Control action response (u) with Z-N method for setpoint tracking

In the $Z-N$ tuning method also the reset control has a much lower overshoot than the other controllers while reaching steady state sooner. However, it has a relatively higher rise time. The PI and PID controllers have higher overshoot and an oscillatory response. Therefore, It can be concluded that the reset controller has tracked the reference input better than the PI and PID controllers tuned with $Z-N$ and automatic tuning methods.

6. ERROR INTEGRAL CRITERION

Measurement of the control system performance has attracted much attention in recent literature on automatic control. There are three commonly used performance indexes including integral square error (ISE), integral absolute error (IAE) and the integral of time multiplied by the absolute value of error $(ITAE)$, respectively.

In order to get a better comparison between the different controllers, IAE and ISE values are applied as the criterions of tracking performance. Indeed, these performance indexes determine that the reset action makes the error increase or decrease.

The criterion of $ITAE$ can be suitably employed to reduce the setting time of the output response. In order to minimize the response error, the criteria ISE or IAE are still a good quality measure for PID controller settings, and they are defined respectively by

$$IAE = \int_0^\infty |e| dt = \int_0^\infty |r_{setpoint} - y_{output}| dt$$
$$ISE = \int_0^\infty e^2 dt = \int_0^\infty (r_{setpoint} - y_{output})^2 dt \qquad (12)$$

TABLE 5
Error Integral Criterion PI, PID, Reset with Z-N, Auto-tune

		PI	PID	$PI-CI$
$Z-N$	IAE	9.07×10^5	9.99×10^5	9.056×10^5
	ISE	7.688×10^8	8.55×10^8	5.691×10^8
$Auto-Tun$	IAE	2.358×10^5	7.42×10^4	4.907×10^5
	ISE	4.66×10^7	1.23×10^7	2.657×10^8

According to the table (5) the following results are obtained, in $Z-N$ method, highest value of IAE

belongs to PID controller. In the reset control, ISE has the lowest value. Also, the ISE value is maximum in PID control. In auto-tune method, PID Controller shows a lower IAE value than the other controllers and reset control has lower IAE value than PID controller. The ISE value is minimum in PID controller with auto-tune method whereas reset controller provides a maximum value of ISE.

The simulations in this paper show the following results:

The reset control has settling time (% 95 step values) 1000 second, but the PI and PID controllers have 1500 second. Thus, reset has a faster behavior. The reset has lower overshoot almost 5% that is 20% smaller than other controllers. According to Fig. 13, control action signal in PID control has 25% undershoot while reset has no undershoot.

As a shown Fig. 14, in $Z-N$ method, the reset has 5% overshoot but other controllers have more overshoot about 90%. Also, reset is faster than PI and PID controllers and its settling time is 900 second. In this case, the time required to reach steady state on output response is 1000 sec in PID and 2500 sec in PI.

Additionally, some of the performance indexes are improved in reset. For example, IAE parameter is reduced more than 10% and ISE is reduced more than 100% in $Z-N$ method.

As a result, we can confirm that reset control improves performance of system.

Effect of reset ratio on the Error integral criterion

According to the table (6), the increase of the reset ratio will rise the values of IAE and ISE. With increment of the reset ratio, the effect of the reset mechanism increases while the effect of the integrator decreases. This effect is more pronounced when the reset ratio is higher than 0.5. It's considered whenever the reset ratio goes above 0.65, the values of the error integral increase more, hence the effect of reset ratio increase leads to increase of ISE and IAE values. Therefore, in order to have a better performance, the reset ratio is chosen below 0.5 and is tuned to give the best output response.

Table 6
Effect of reset ratio on the Error integral criterion

ρ_{reset}	0.1	0.3	0.6	0.9
$IAE \times 10$	5.16	6.55	11.1	28.2
$ISE \times 10$	2.71	3.21	5.4	14

7. CONCLUSION

In this paper, a controller is designed for a chemical process with PI, PID and reset control methods. Moreover, the reset control systems are applied to a CSTR temperature model. Firstly, a model of the plant is obtained and then PI, PID controllers are designed with two different methods (automatic and $Z-N$). Next, the reset control is replaced with them. Finally, the reset parameters are tuned to improve the results. The output responses are compared from disturbance rejection and set-point tracking point of view. The results show that applying a PID controller in the $Z-N$ method can be useful, but the reset control can be a suitable alternative for the existing controllers. Moreover, the reset action improves the performance of system in set-point tracking.

Table 7
Parameters of the reactor

Parameters	Description	Value
F	Reactant's flow rate	0.007($m^3.s^{-1}$)
V	tank volume	3.5(m^3)
ρ	Density of the liquid	1089($kg.m^{-3}$)
C_p	Capacity of temperature	3348 ($J.kg^{-1} {}^\circ C^{-1}$)
U	Heat transfer coefficient	715($W.m^{-2} {}^\circ C^{-1}$)
A	Heat transfer area	22(m^2)
k_u	Gain of actuating	0.01
τ_v	Time constant of actuating	12 (s)
k_t	Sensor gain	1
τ_t	Time constant of sensor	45(s)
$\overline{\Theta}_i$	Temperature of input liquid	$38^\circ C$
$\overline{\Theta}$	Temperature of output liquid	$65^\circ C$

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