Bidding Strategy in Spot Markets with Definition of a New Market Power Index by Using Conjectural Variation

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**ABSTRACT**

In this paper, the concept of Conjectural Variation (CV) is used to specify optimal generation decision for generation companies (Gencos). The conjecture of Genco is defined as its belief or expectation about the reaction of rivals to change of its output. Using CV method, each Genco has to learn and estimate strategic behaviors of other competitors from available historical market operation data. Therefore, accuracy of generation decision depends on the accuracy of estimating other competitors' decision within CV context.

In this paper, adjusted Lerner index is used to improve the accuracy of estimating CV parameter. In electricity market, the adjusted Lerner index can be directly computed using price, market shares, marginal cost and industry elasticity of demand. It must be noted that due to repeated power market, Gencos need to modify their behavior over time. In response to this need, dynamic learning is considered in case studies which improve results.

**KEYWORDS**: Power market, Conjectural Variation, Adjusted Lerner index, Bidding strategy

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**1. INTRODUCTION**

From early years of emerging electrical industry until early 80’s, governments were in charge of providing electrical energy demand. In the last few years, traditional cost-driven trends of energy supply have been revised to maximize unexploited capabilities of traditional power generation, transmission and distribution via deregulating unilateral monopoly of governments in competitive environments. In a market-based power supply system, energy is regarded as commodity in which GENCOS trade it to make profit. In other hand, limited number of suppliers, let GENCOS to exercise market power; decrease or increase of bidding quantity of a generation company which possesses market power, can change market-clearing price and production of other GENCOS. In such conditions, obtaining optimum bidding in an oligopolistic power market is of great importance.

A group of studies used ideal model to investigate GENCOS bidding. In ideal model, it is assumed that during bidding, GENCOS are completely aware of bidding and marginal costs of counterparties. However, counterparties’ bidding data are confidential and is not available to public. Hence, recent studies focused on developing GENCOS bidding in presence of uncertainties of the market [1]. Optimal bidding methods can be divided into three main categories [2]: methods based on Market Clearing Price (MCP) forecasting, methods based on modeling bidding curves of parties and game theory based methods [3, 4]. In MCP forecasting, estimation is based on whether neural networks or time series approach [5-7]. In order to forecast with MCP, it is crucial to have bidding curves of all GENCOS. Second group of studies utilize theory of probability to optimize bidding in presence of uncertainties. In the third group, investigators develop optimum bidding
by means of game theory. In a deregulated market, GENCOs possess oligopoly to supply electrical energy [8]. Game theory analyzes economic behaviors of GENCOs to obtain market equilibrium and consequently obtaining optimal bidding strategy for proposed GENCO [9] which proposed an analytical solution for bidding strategy problem of transmission-constrained GenCos in an hour-ahead electricity market. Here, modeling of bidding strategy problem is under a supply function equilibrium model. Each GenCo ignores the reaction of other rivals when maximizes its own profit, while it is wrong due to the profit of treats as a function of its own bidding strategy and the bidding strategies of other rivals. In the proposed method it is considered these reactions by considering the historical market data. Many equilibrium based studies used conjectural variation (CV) method to model strategic interactions of GENCOs [10]. In conjectural variation, strategic interactions are based on estimating the reaction of the counterparties to a conjectural change in production of an assumed supplier. This method was first proposed by Frisch [11] and firstly Song et al. applied this method to model bidding behavior of GENCOs in oligopolistic power supply market. Afterwards, many studies inspired from CV in which optimal bidding of GENCOs were calculated in predefined conjectural variation in static conditions. Estimating conjectural variations is not simple task because of complication and dynamics of power market. In other hand, generation companies participating in day-ahead market or repetitive market [12] can estimate conjectural variation more precisely by learning from historical data, hence in [13], learning method based on CV was practiced to obtain bidding strategy.

This paper analyzes bidding strategies of generation companies in repetitive market via conjectural variation concept. Uncertainties in electricity market cause that the suggested Nash equilibrium based on CV method deviate from real market equilibrium.

Even though GENCOs access same historical data, each generation company has its specific interpretation of market history; therefore exact market equilibrium estimation is not possible. This paper proposes conjectural variation based on adjusted Lerner index to achieve more realistic bidding. Additionally for learning repetitive market GENCOs, dynamic learning approach was applied. In the second section of this paper Generation Company bidding problem is modeled. The third section considers dynamic learning of GENCOs. In the fourth part, Generation Company competition is modeled and finally results are discussed.

2. OPTIMAL BIDDING QUANTITY OF GENCOs

Assume that N generation companies compete in an oligopolistic power market to supply system load D for next hour, and transmission capacity is large enough. Market inverse demand function would be:

\[ P = A - K.D = A - K.(q_i + q_{-i}) \]  \hspace{1cm} (1)

A and K are intercept and slope of the demand curve, respectively. A and K are positive values. Market clearing mechanism for such market is in such way that GENCOs propose their bidding quantity, then ISO receives supply bidding all GENCOs and by intersecting it with demand curve, obtains market equilibrium. Since market clearing price and bidding quantity of each generation company is influenced by competitions in the market, each GENCO attempts to maintain its bidding quantity in such a way that achieve maximum profit. Cost function of \( i^{th} \) GENCO is:

\[ C_i(q_i) = \frac{1}{2}C_iq_i^2 + b_iq_i + a_i \]  \hspace{1cm} (2)

In which \( c_i>0, b_i>0 \) and \( a_i>0 \); in such conditions, marginal costs for \( i^{th} \) Generation Company is equal to:

\[ MC_i(q_i) = c_iq_i + b_i \]  \hspace{1cm} (3)

Therefore, the problem of obtaining optimal bidding quantity for \( i^{th} \) GENCO is as below:

\[ \text{Max } q_i; \pi_i = p \cdot q_i - C_i(q_i) \]

\[ q_{i,\text{min}} \leq q_i \leq q_{i,\text{max}} \]  \hspace{1cm} (4)

In which \( \pi_i \) is profit of \( i^{th} \) Generation Company, \( p \) is market clearing price, \( C(q_i) \) is cost function, \( q_{\text{min}} \) and \( q_{\text{max}} \) are minimum and maximum production of the \( i^{th} \) company, respectively. In order to calculate optimal bidding quantity:

\[ \frac{\partial \pi_i}{\partial q_i} = \frac{\partial p}{\partial q_i} \cdot q_i + p - MC_i(q_i) = 0 \]  \hspace{1cm} (5)

Since

\[ D = \sum_{i=1}^{N} q_i = q_i + \sum_{j=1, j \neq i}^{N} q_j = q_i + q_{-i} \]  \hspace{1cm} (6)

In order to calculate \( \frac{\partial p}{\partial q_i} \), one can write:

\[ \frac{\partial p}{\partial q_i} = \frac{\partial p}{\partial D} \cdot \frac{\partial D}{\partial q_i} = \frac{\partial p}{\partial D} \cdot \left( 1 + \frac{\partial q_{-i}}{\partial q_i} \right) = \frac{\partial p}{\partial D} \cdot \left( 1 + \frac{CN_i}{q_i} \right) \]  \hspace{1cm} (7)

In which conjectural variations \( \frac{\partial q_{-i}}{\partial q_i} \) demonstrates counterparties’ reaction to conjectural variation in bidding quantity of \( i^{th} \) Generation Company. According to equations (5-7), optimal bidding quantity for \( i^{th} \) Generation Company equals to:

\[ q_i^* = \left( \frac{CN_i}{2(\partial p/\partial D)} \right) \]  \hspace{1cm} (8)
\[ q_i = \frac{A - K \cdot q_{i-1} - b_i}{K \cdot (2 + CV_i) + c_i} \]  

(8)

It should be noted that in assumed market, each GENCO is also trying to achieve maximum profit. According to game theory, competition of generation companies contributing to a non-cooperative game under oligopolistic conditions leads to Nash equilibrium. Such equilibrium is stable because all generation companies achieve maximum profit. In other words, if a GENCO does not take its bidding strategy according to Nash equilibrium, it will definitely suffer loss of profit. Hence, in order to obtain optimal bidding quantity of generation companies in Nash equilibrium, can write:

\[
q^* = \left\{ \begin{array}{l}
q^*_1 = \frac{A - K \cdot q^*_0 - b_1}{K \cdot (2 + CV_1) + c_1} \\
q^*_N = \frac{A - K \cdot q^*_N - b_N}{K \cdot (2 + CV_N) + c_N}
\end{array} \right.
\]  

(9)

\[
q_{i,\text{min}} \leq q_i \leq q_{i,\text{max}}
\]

\[
\sum_{i=1}^{N} q_i = D \\
p = A - K \cdot D
\]

In which \( q^* \) is optimal bidding quantity vector of GENCOs in Nash equilibrium. It must be noted that Nash Equilibrium of \( q^* \) is Proportional to CV\(^*\). In other words, gained profit is proportional to accuracy of estimation of counterparties’ behavior. In addition, insight of companies contributing in a competitive market about their counterparties’ behavior changes with time. Therefore, attaining maximum profit in a repetitive market depends on learning from historical data of the market and dynamic correction of the insight about counterparties’ behavior.

3. LEARNING DYNAMIC BIDDING

In a repetitive market of electric power supply, bidding strategies of GENCOs are based on historical data. Since attaining optimal strategy requires accurate estimate of the CV value, the CV-based dynamic learning modifies the company insight for current period by investigating past periods data; finally estimated CV matches real behavior of counterparties.

Therefore, in dynamic learning method, optimal bidding quantity of \( i^{th} \) GENCO in the market at \( t^{th} \) period based on counterparties’ behavior and also CV at \((t-1)^{th}\) time interval are obtained in such a way that:

\[
q_i^t = \frac{A - K \cdot q_{i-1}^{t-1} - b_i}{K \cdot (2 + CV_{i}^{(t-1)}) + c_i}
\]  

(10)

To extract CV value from historical data, one can utilize adjusted Lerner index; \( \beta_i \) is equal to \( 1 + CV_i \) (eq. (7)). Adjusted Lerner index \( \beta_i \) equals to [14], [15]:

\[
\beta_i = \frac{a \cdot p - MC_i}{p}
\]  

(11)

In which \( S_i \) is market share of \( i^{th} \) GENCO from system market power, \( P \) is market clearing price and \( a \) demonstrates demand elasticity to change in price.

\[
\alpha = -\frac{\partial D}{\partial p} = -\frac{P}{D} \frac{\partial D}{\partial p}
\]  

(12)

Hence, for performing learning process one can write demand elasticity at time \( t \) based on historical date as:

\[
\alpha_t = -\frac{p_{t-1}}{D_{t-1}} \frac{1}{K}
\]  

(13)

To estimate market share of the \( i^{th} \) GENCO based on adjusted Lerner index:

\[
S_{li} = \frac{\beta_{i(t-1)} q_{i(t-1)}}{D_{t-1}}
\]  

(14)

Based on eq. (1), market price at time period of \( t \) is:

\[
p_t = A - K \cdot D_t
\]  

(15)

According to definition, marginal cost of \( i^{th} \) GENCO at time \( t \) is:

\[
MC_{it} = c_i q_{it} + b_i
\]  

(16)

After assessment of adjusted Lerner index according to historical data, proper learning process after different iterations can led to acceptable prediction of optimal bidding quantity for \( i^{th} \) GENCO. Learning process was performed via below equation according to [13]:

\[
\beta_{it} = \left\{ \begin{array}{l}
\beta_{i(t-2)} \frac{p_{t-2} - MC_{i(t-2)}}{S_{li(t-2)}} \cdot |q_{i(t-2)} - q_{i(t-3)}| \geq \varepsilon \\
\beta_{i(t-1)} \cdot |q_{i(t-2)} - q_{i(t-3)}| \leq \varepsilon
\end{array} \right.
\]  

(17)

In which \( \varepsilon \) is a small positive value and expresses demand elasticity.

Nash equilibrium is obtained when during learning process, adjusted Lerner index converges to reality.
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\[ q_i^* = \begin{cases} 
A - K \cdot q_{i-1} - b_i \\
K \cdot (1 + \beta_i^+) + c_i 
\end{cases} \]
\[ q_N^* = \frac{A - K \cdot q_N - b_N}{K \cdot (1 + \beta_N^+) + c_N} \]
\[ q_{i,\text{min}} \leq q_i \leq q_{i,\text{max}} \]
\[ \sum_{i=1}^{N} q_i = D \]
\[ p = A - K \cdot D \]

(18)

4. NUMERICAL SIMULATIONS

This section discusses estimation of optimal bidding quantity in duopoly market and also oligopolistic market via the proposed method. Additionally, Nash equilibrium value based on [13] is obtained to check validity of the results.

A. Duopoly market

In order to investigate the proposed method and further illustration of the learning procedure, this section deals with estimation of optimal bidding quantity of companies in a simple duopoly market, considering 4 case studies.

Market demand curve assumed as \( p=35-0.018D \) and company's cost function data are shown in table 1. Learning was performed for 300 iterations. Initial value of CV and adjusted Lerner index assumed -0.8 for the first three iterations and initial bidding quantity of generation companies 1 and 2 were 446 (MW) and 709.98 (MW), respectively [13].

<table>
<thead>
<tr>
<th>GENCO</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>Min Product</th>
<th>Max Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0.025</td>
<td>0</td>
<td>800</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.75</td>
<td>0.0175</td>
<td>0</td>
<td>800</td>
</tr>
</tbody>
</table>

B. First Case Study

In this case, there is no competition in the market. As mentioned before - and simulation results confirm - different game theory models are special cases of CV method [2] and also the proposed method. This case is equivalent to CV=-1 in CV method. It is also corresponding to \( \beta \) values (market power index) equal to zero in the proposed method. Results are shown in tables 2 and 3.

C. Second Case Study

This case is equivalent to model of Stackelberg in classic game theory. Here, first GENCO is assumed to be "Leader" and obtains its CV by learning process. Second company supposed to be "Follower" which ignores strategic reactions of the counterparties for future production decisions (or CV=0). It is also assumed that first company can exploit historical data of the market (its own bidding historical data or the load historical data) for its bidding strategy. Simulation data for both dynamic CV and the proposed method are shown in tables 1 and 2.

Precision of the results of market equilibrium method is revealed by comparing results from CV method with results of [13].

Comparing results of CV simulation and the proposed method, it is shown that market clearing price is increased from 18.72 $/MW (by CV-based method) to 18.97 $/MW in the proposed method; therefore profit is increased. In comparison of the proposed method with the first case study, profit of the first company is increased from $2497 to $4420 and for the second company it was improved from $4410$ to $6296$, respectively.

D. Third Case Study

In this case, in contrast to the previous case, assumed that the first company is "Follower" and the second one is the "Leader" company which utilizes historical data of the market for bidding strategy. By comparing tables 2 and 3, it is revealed that market clearing price (MCP) increased from 18.5 $/MW for CV-based method to 18.28 $/MW for the proposed method.

E. Fourth Case Study

Here, it is assumed that both generation companies employ bidding strategy learning algorithms. Results of simulation with CV-based method are shown in table 2. Comparing these results with [13] confirms the accuracy of results.

<table>
<thead>
<tr>
<th>Case Study</th>
<th>---</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price($/MW)</td>
<td>---</td>
<td>14.17</td>
<td>18.72</td>
<td>18.51</td>
<td>17.82</td>
</tr>
<tr>
<td>Total Production (MW)</td>
<td>---</td>
<td>1157</td>
<td>904</td>
<td>916</td>
<td>954.4</td>
</tr>
<tr>
<td>GENCO 1</td>
<td>CV</td>
<td>-1</td>
<td>-0.33</td>
<td>0</td>
<td>-0.38</td>
</tr>
<tr>
<td>Production (MW)</td>
<td>446.9</td>
<td>425.7</td>
<td>360.7</td>
<td>410</td>
<td></td>
</tr>
<tr>
<td>Profit ($)</td>
<td>2497</td>
<td>4430</td>
<td>3969</td>
<td>3975</td>
<td></td>
</tr>
<tr>
<td>GENCO 2</td>
<td>CV</td>
<td>-1</td>
<td>0</td>
<td>-0.29</td>
<td>-0.33</td>
</tr>
<tr>
<td>Production (MW)</td>
<td>709.9</td>
<td>478.3</td>
<td>555.3</td>
<td>544.4</td>
<td></td>
</tr>
<tr>
<td>Profit ($)</td>
<td>4410</td>
<td>6118</td>
<td>6609</td>
<td>6156</td>
<td></td>
</tr>
</tbody>
</table>

Convergence speed was a concern in [13] in which MCP converged after 25 iterations in [13]. In this study, applying sensitivity analysis for CV-based method [16], convergence speed improvement has
been obtained and also MCP converged after 10 iterations (as shown in Fig. 1). Results for simulating this case with the proposed method are given in table 3. In comparison with CV-based method, the proposed method increased gained profits of GENCOs. Also, MCP is improved from 17.82 $/MW in CV-based method to 19.92 $/MW for the proposed method. As a result, profit of the first company is increased from 3975 $ by CV-based method to 4185.5 $ for the proposed method. Second company’s profit is improved from 6156 $ (CV-based) to 7010 $ via the proposed method which is converged after 19 iterations.

In this simulation, learning process of companies obey eq. 17. Initial $\beta$ value assumed to be 0.01 for all companies and initial bidding quantity presumed to be equal to supply value in a perfect competitive market. Simulation results for both CV-based and the proposed methods are shown in table 5. According to table 5, gained profits are increased so that market clearing price is increased from 17.47 $/MW in CV-based method to 17.88 $/MW in the proposed method. All generation companies gain more profit with less production via the proposed method rather than dynamic CV-based method.

### Table 3
**Simulation Results for the Proposed Method in Duopoly Market**

<table>
<thead>
<tr>
<th>Case Study</th>
<th>Price ($/MW)</th>
<th>Total Production (MW)</th>
<th>Profit ($)</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENCO 1</td>
<td>17.82</td>
<td>446.9 986</td>
<td>2497</td>
<td>0</td>
</tr>
<tr>
<td>GENCO 2</td>
<td>17.88</td>
<td>709.9 1873</td>
<td>4410</td>
<td>1</td>
</tr>
</tbody>
</table>

![Figure 1: MCP convergence speed in dynamic CV-based method in 4 different cases](image)

### F. Oligopolistic power market with six players

In this section CV learning algorithm and the proposed learning algorithm are applied to obtain optimal bidding quantity for 6 generation companies in an oligopolistic power market. Each generation company utilizes historical data while models other counterparties as a virtual overall competitor. Coefficients of cost function for generation companies are shown in table 4.

In this simulation, learning process of companies

### Table 4
**Coefficients for Generation Companies Cost Functions**

<table>
<thead>
<tr>
<th>GENCO</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>Min Product</th>
<th>Max Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0.02</td>
<td>0</td>
<td>800</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.75</td>
<td>0.0175</td>
<td>0</td>
<td>800</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>3</td>
<td>0.025</td>
<td>0</td>
<td>800</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>3</td>
<td>0.025</td>
<td>0</td>
<td>800</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>1</td>
<td>0.0625</td>
<td>0</td>
<td>800</td>
</tr>
<tr>
<td>6</td>
<td>3.25</td>
<td>1</td>
<td>0.00834</td>
<td>0</td>
<td>800</td>
</tr>
</tbody>
</table>

### Table 5
**Simulation Results for the Proposed Method in Duopoly Market**

<table>
<thead>
<tr>
<th>Price($/MW)</th>
<th>Total Production(MW)</th>
<th>CV-based Method</th>
<th>Proposed Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.47</td>
<td>3595</td>
<td>680.6</td>
<td>649</td>
</tr>
<tr>
<td>17.88</td>
<td>3490</td>
<td>5896</td>
<td>6093</td>
</tr>
<tr>
<td>GENCO 1</td>
<td>CV</td>
<td>0.29</td>
<td>1.154</td>
</tr>
<tr>
<td>GENCO 2</td>
<td>CV</td>
<td>0.8</td>
<td>0.01</td>
</tr>
<tr>
<td>GENCO 3</td>
<td>CV</td>
<td>0.52</td>
<td>1.268</td>
</tr>
<tr>
<td>GENCO 4</td>
<td>CV</td>
<td>0.52</td>
<td>1.268</td>
</tr>
<tr>
<td>GENCO 5</td>
<td>CV</td>
<td>0.81</td>
<td>1.646</td>
</tr>
<tr>
<td>GENCO 6</td>
<td>CV</td>
<td>0.8</td>
<td>0.01</td>
</tr>
</tbody>
</table>

### 5. Conclusion

In this paper, authors utilized conjectural variation method to obtain optimal bidding quantity for generation companies participating in an oligopolistic power market. Uncertainties in behavior of counterparties cause exact value of CV to be unknown.
Therefore, authors suggested applying adjusted Lerner index for dynamic learning of generation companies. Analysis of simulation results and comparing them with previous studies confirms validity of the method. Furthermore, this method improved convergence speed. Transmission security constraints impose decision uncertainty in optimal bidding quantity; therefore by extending the proposed method and applying dynamic learning, Nash equilibrium for security-constrained bidding issue would be obtained.

REFERENCES


BIOPGRAPHIES

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